Observation of a Transition in the $\nu = 5/2$ Fractional Quantum Hall State at Large Landau Level Mixing

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We have studied the $\nu = 5/2$ fractional quantum Hall state in the regime of extremely low densities, for which the Landau level mixing parameter κ spans the so far unexplored $2.52 < \kappa < 2.82$ range. In the vicinity of $\kappa = 2.6$ an unexpectedly large change in the density dependence of the energy gap is observed which is suggestive of a transition in the $\nu = 5/2$ state. Origins of such a transition are discussed, including the possibility of a topological phase transition.

The understanding of the quantum fluids with topological degrees of freedom is an important theme in contemporary condensed matter physics. Such topological quantum fluids are thought to form in systems such as two-dimensional electron gases (2DEGs), topological insulators, and topological superconductors [1–3]. Interest in topological quantum fluids has been fueled by the prediction of non-Abelian quasiparticles which may be harnessed for decoherence-free quantum computation [1–6].

The energy spectrum of a 2DEG subjected to a magnetic field B consists of a set of equally spaced and degenerate energy levels called the Landau levels. The fractional quantum Hall state (FQHS) developing when two such energy levels are completely filled and the third is half filled, i.e. the FQHS at Landau level filling factor $\nu = 5/2$, is one example of such a topological quantum fluid. The work of Moore and Read advanced the Pfaffian description of this state [7]. Besides the Pfaffian other alternative candidate wavefunctions [8–12], such as the anti-Pfaffian solution [8, 9], have been proposed. An ongoing intense experimental effort is not yet able to unambiguously discriminate between these proposals [13–17].

Due to their very similar energy scales, proposed ground states at $\nu = 5/2$ are expected to be in close competition [18] and, therefore, the delicate energy balance can be tipped by a perturbation such as Landau level mixing (LLM). LLM is the coupling between different Landau levels caused by the Coulomb interaction and it is quantified by the dimensionless parameter $\kappa = E_c/\hbar\omega_C$ [19]. Here E_c is the Coulomb energy and $\hbar\omega_C$ the cyclotron energy. It is well recognized that by generating effective three-body terms in the interaction potential, LLM profoundly affects the Pfaffian and anti-Pfaffian states at $\nu = 5/2$ [8, 9, 20–30]. Furthermore, LLM was recently proposed to induce a transition between the Pfaffian and the anti-Pfaffian [8, 9, 25, 26]. Since the Pfaffian and anti-Pfaffian states are topologically distinct, a transition between them would be a prototypical topological phase transition [8, 9]. Experiments on the $\nu = 5/2$ FQHS, which so far have accessed LLM parameters ranging from moderate values $\kappa \approx 1$ to values as as high as $\kappa = 2.2$, have found no evidence of a Pfaffian to anti-Pfaffian transition [31–34].

Motivated by these ideas we undertook a study of the energy gaps of the $\nu = 5/2$ FQHS in the so far unexplored regime of strong LLM with $\kappa > 2.5$. The LLM is tuned by the well known technique of changing the electron density. In this work we report the observation of a sharp change in the density dependence of the energy gap of the $\nu = 5/2$ FQHS, a change which is suggestive of a transition in the $\nu = 5/2$ FQHS. This sharp change occurs in the vicinity of $\kappa = 2.6$ and may be a first evidence of a transition between two different gapped ground states at $\nu = 5/2$, such as a topological phase transition between the Pfaffian and anti-Pfaffian. Other possible origins of the observed anomalous density dependence are also discussed.

We studied a 65 nm wide symmetrically doped GaAs/AlGaAs quantum well sample with the density of $n = 6.13 \times 10^{10}$ cm^{-2} and the corresponding mobility of $\mu = 9.1 \times 10^6$ cm^2/Vs. Heavily doped samples, such as ours, are often not responsive to the established technique of tuning the density by electrostatic gating. Instead of gating we use a low intensity illumination technique on an ungated sample. The preparation of the 2DEG starts with an illumination with a red light-emitting diode (LED) at 10 K with 1 mA excitation. As seen from Fig.1a, this sets the density of our sample to $n = 6.13 \times 10^{10}$ cm^{-2}. To decrease the density, we apply a low excitation of the order of $1\mu A$ to the same LED for about 5 minutes while keeping the sample temperature close to 10 mK. As shown in Fig.2a, the density reduction obtained as a result of successive low temperature irradiations scales with the product of the LED excitation current and the time of illumination, a product we call the integrated light intensity. As the density is reduced, the carrier mobility decreases from its peak value of $\mu = 9.1 \times 10^6$ cm^2/Vs. This is shown on Fig.2b. While the illumination of samples has been an accepted technique for state preparation and it has been demonstrated that it may reduce the electron density [35], to the best of our knowledge this is the first time the low intensity illumination has been used for a controlled density ma-

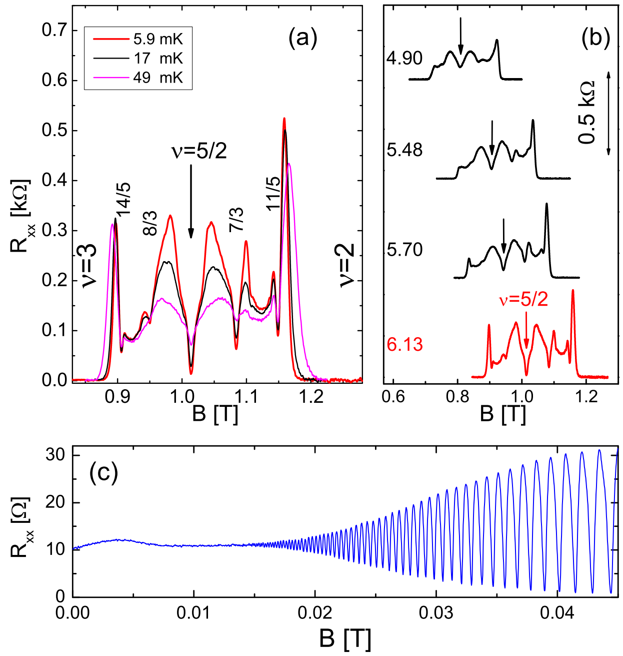

FIG. 1. (a) The magnetoresistance at three different temperatures at n=6.13×10^{10} cm^{-2}. The integer and fractional quantum Hall states are marked by their filling factor ν. (b) The magnetoresistance at T = 5.9 mK at several different electron densities. The labels are densities in units of 10^{10} cm^{-2}. Arrows mark the position of the $\nu=5/2$ FQHS. (c) Shubnikov-de Haas oscillations at n=6.13×10^{10}cm^{-2} and $T=5.9$ mK.

nipulation.

In Fig.1a we show the longitudinal magnetoresistance R_{xx} in the vicinity of $\nu=5/2$ at three selected temperatures T and at the highest achieved electron density $n=6.13\times10^{10}$ cm^{-2}. This is the lowest density to date at which the energy gap of the $\nu=5/2$ FQHS has been studied [34]. The magnetic field B in Fig.1a is such that the Landau level filling factor ν spans the $2<\nu<3$ range, commonly referred to as the lower spin branch of the second Landau level. Here the filling factor is obtained from $\nu=hn/eB$, where h is Planck's constant and e is the elementary charge. At $\nu=5/2$ we observe a deep magnetoresistance minimum of less than 15 Ω. Besides the FQHS at $\nu=5/2$, we also observe strong FQHSs at $\nu=7/3$ and 11/5, and there are indications of developing FQHSs at $\nu=8/3$, and 14/5. Furthermore, in Fig.1a we notice three peaks in R_{xx} at $B=1.10$, 1.04, and 0.98 T. We associate these peaks with precursors at temperatures above the onset temperature of the reentrant integer quantum Hall states $R2a$, $R2b$, and $R2c$, respectively [36].

In wide quantum well samples, such as ours, one needs to consider the possibility of populating the second electric subband [33, 37, 38]. However the absence of beating in the Shubnikov-de Haas oscillations, shown in Fig.1c, indicates that in our sample only the lowest electric subband is occupied at the highest density. At lower densities population of the second subband is not expected.

The activation energy gap Δ is extracted from the temperature dependence of the magnetoresistance using the $R_{xx}\propto\exp(-\Delta/2T)$ relation. For achieving a good thermal contact between the 2DEG and the thermal bath we used a He-3 immersion cell and for thermometry we employed quartz tuning fork viscometry [39]. The Arrhenius plots of the magnetoresistance at $\nu=5/2$ for several different electron densities are shown in Fig.3a and Fig.3b. At the highest prepared density $n=6.13\times10^{10}$ cm^{-2}, the gap of $\nu=5/2$ FQHS is $\Delta_{5/2}$=80 mK. A similar measurement of the $\nu=7/3$ FQHS (not shown) results in $\Delta_{7/3}$=27 mK. The energy gap at this density of the $\nu=7/3$ FQHS is, therefore, a factor of 3 smaller than that of the $\nu=5/2$ FQHS. This result is surprising since in numerous samples the energy gaps at $\nu=7/3$ and $\nu=5/2$ are comparable [31, 34, 37, 40–45]. For example, the gaps of these two states in a sample of low density $n=8.3\times10^{10}$ cm^{-2} are $\Delta_{5/2}=88$ mK and $\Delta_{7/3}=81$ mK [34]. Exceptions are the samples discussed later in our manuscript which are not in the strictly two-dimensional limit, i.e. which have two electric subbands occupied [33, 38].

As the density is decreased using the cold illumination technique, both the $\nu=5/2$ and 7/3 FQHS become more fragile. This is seen in In Fig.1b where we show the evolution of R_{xx} with the density, taken at $T=5.9$ mK. Activated T-dependence of the magnetoresistance at $\nu=5/2$ shown in Fig.3a and Fig.3b results in suppressed energy gaps. Such a suppression of the $\nu=5/2$ FQHS with a decreasing density, and hence increasing LLM, is expected [21, 22, 27] and has been experimentally observed [34]. The density dependence of the energy gaps of $\nu=5/2$ and $\nu=7/3$ FQHSs is captured in Fig.4. At $\nu=5/2$ the activated behavior per-

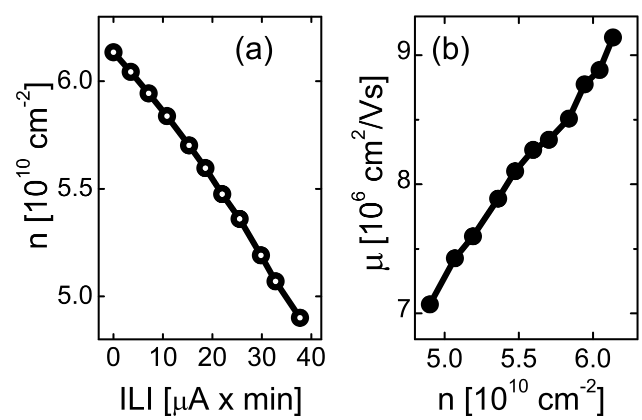

FIG. 2. (a) Te electron density n as a function of the integrated light intensity (ILI). (b) The mobility μ of the 2DEG as a function of the density.

sists to a density as low as $n = 4.90 \times 10^{10}$ cm^{-2}, while at $\nu = 7/3$ only down to $n = 5.60 \times 10^{10}$ cm^{-2}. The top scale of Fig.4 shows the LLM parameter $\kappa = E_c/\hbar\omega_C$ calculated at $\nu = 5/2$, where $E_c = e^2/4\pi\epsilon l_B$ is the Coulomb energy, $l_B = \sqrt{h/eB}$ the magnetic length, and ϵ the dielectric constant of the GaAs host [19]. We note that at a given filling factor κ scales with $1/\sqrt{n}$ [19]. Owing to the low electron densities reached, the LLM parameter at $\nu = 5/2$ in our sample is unprecedentedly large and as seen in Fig.3, it spans the $2.52 < \kappa < 2.82$ range.

The most salient feature of our data in Fig.4 is the change in the slope of the $\Delta_{5/2}$ versus n curve which is apparent in a markedly positive curvature. At the highest densities $\Delta_{5/2}$ decreases very steeply with a decreasing n. In contrast, at the lowest densities $\Delta_{5/2}$ has a more gentle slope. The change in the slope of the $\Delta_{5/2}$ versus n data occurs close to the density of 5.8×10^{10} cm^{-2} or $\kappa = 2.6$. Fitting the low and high ends of our $\Delta_{5/2}$ versus n plot with straight lines, we find that in Fig.4 the slope $\partial\Delta_{5/2}/\partial n$ changes by factor of 10. Remarkably, this substantial change in slope takes place as the density is changed by less than 5%. This abrupt change in the slope over a narrow density range is suggestive of a transition between two different gapped ground states at $\nu = 5/2$ induced by the changing density. In the following, we explore the possible origins of this unexpected behavior of the $\nu = 5/2$ FQHS.

We first compare the density dependence of the gap at $\nu = 5/2$ of our sample to that of other density tunable samples from the literature. Prior data is taken at considerably higher electron densities, $n > 1.3 \times 10^{11}$cm^{-2} [31], $n > 1.8 \times 10^{11}$cm^{-2} [32], and $n > 2.6 \times 10^{11}$cm^{-2} [33]. Data from Refs.[33] are quite different from other data because of the precipitous collapse of $\Delta_{5/2}$ with an increasing density. This collapse of the gap was associated with the population of the second electric subband [29, 33]. A positive curvature of the $\Delta_{5/2}$ versus n curve we measure is not seen in Refs.[31, 32], but it is present in the low density end of the curves from Ref.[33] at densities at least 5 times higher. There are, however, three important differences between our data and that from Ref.[33]. First, our experiment and that of Ref.[33] are at the opposite extremes of LLM strength: in our sample the LLM at $\nu = 5/2$ is the largest to date whereas LLM in Ref.[33] is among the weakest. Second, as we have already discussed, the absence of beating in the Shubnikov- de Haas oscillations in our sample indicates that, contrary to samples from Ref.[33], the second electric subband is not populated in our sample. Third, the relationship between the 5/2 and 7/3 FQHSs is very different. In Ref.[33] in the vicinity of populating the second electric subband 7/3 is dramatically enhanced. Not only we do not see such a behavior, but also the gap at $\nu = 7/3$ in our sample is considerably lower than that at $\nu = 5/2$. In addition, when populating the second subband an enhanced scattering is expected to decrease the mobility [46, 47], which is absent in Fig.2b. To summarize, we think that the anomalous behavior of $\nu = 5/2$ FQHS in our sample is most likely unrelated to the population of the second electric subband observed in Ref.[33] and, therefore, the markedly positive curvature of $\Delta_{5/2}$ versus n seen in Fig.4 has a different origin.

One possible interpretation of our data is a transition from fully to partially polarized state at $\nu = 5/2$ as the density is lowered. Experiments probing the spin polar-

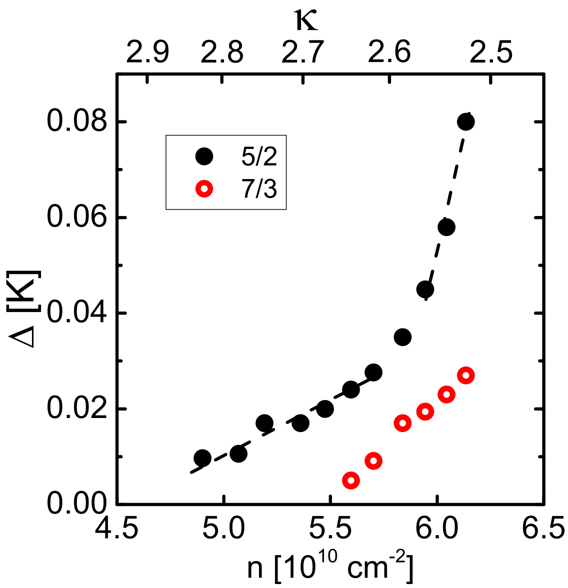

FIG. 4. Dependence of the energy gaps of the $\nu = 5/2$ and $\nu = 7/3$ FQHSs on the density. The top scale shows the LLM parameter κ calculated at $\nu = 5/2$. Dashed lines are linear fits to the data.

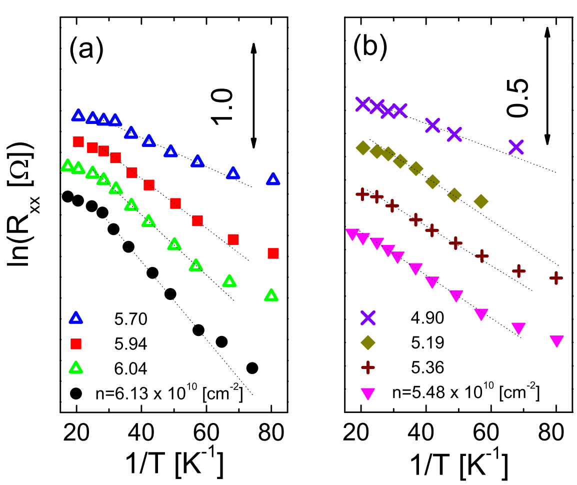

FIG. 3. Arrhenius plots for R_{xx} at $\nu = 5/2$ filling factor at several representative densities. Each consecutive dataset is shifted for clarity.

ization at $\nu = 5/2$ have not found any evidence of a spin transition [32, 47–52]. However, these experiments have been done on samples of substantially higher densities, and hence larger Zeemann splitting, than ours [32, 47–52]. We thus have to carefully examine the possibility of a spin transition in our sample of very low density. The hallmark of a spin transition in a FQHS in transport measurements is a pronounced minimum in the energy gap of a FQHS as the ratio of Zeeman and cyclotron energy scales is varied [47, 53–55]. In the second Landau level such a behavior has recently been seen at $\nu = 8/3$ [53] and at $\nu = 12/5$ [54]. However, as seen in Fig.4, our data does not exhibit such a pronounced minimum of the gap. A spin transition at $\nu = 5/2$ in our sample is therefore unlikely.

A LLM induced phase transition between two distinct gapped ground states at $\nu = 5/2$ is another interpretation of our data. LLM is known to be the leading generator of three-body terms $V^{(3)}$ in the effective interaction and the sign of this three-body potential determines whether the preferred ground state is the Pfaffian or the anti-Pfaffian. However, due to difficulties arising from a large Hilbert space, the inclusion of LLM in numerical calculations remains a formidable task [20–30]. Most recently it was found that at $\nu = 5/2$ the anti-Pfaffian is stabilized at moderate LLM $\kappa \approx 1$ [28, 29] and it was argued that, as the LLM is tuned, a transition between the Pfaffian and the anti-Pfaffian is expected [25, 26]. However, the exact value of κ_{crit} at such a transition is unknown. We may obtain an estimate from a recent calculation of the $m = 3$ term of this potential $V^{(3)}_{3,3/2} \approx -0.0147\kappa + 0.006\kappa^2$ [26] which vanishes at $\kappa_{crit} \approx 2.5$. The abrupt change in the slope of $\Delta_{5/2}$ versus n curve shown in Fig.4 in the vicinity of $\kappa = 2.6$ is tantalizingly close to the above estimate. We note, however, that in lack of knowledge of the contributions from particle-hole symmetry breaking terms other than $V^{(3)}_{3,3/2}$ [26, 27], our estimate of κ_{crit} remains quite crude. Nonetheless, if a Pfaffian to anti-Pfaffian transition does indeed occur, we expect it at large values of κ and hence the observation of a transition in the $\nu = 5/2$ FQHS close to $\kappa = 2.6$ is consistent with this expectation. Therefore a topological phase transition, such as the Pfaffian to anti-Pfaffian transition, remains an exciting possible interpretation of our data.

The observed anomalous density dependence of $\Delta_{5/2}$ may also be caused by the effects of the disorder. In a recent publication, following the analysis proposed in Ref.[21], we extracted the intrinsic gap and the disorder broadening for the $\nu = 5/2$ FQHS in several samples [34]. In the absence of a measurable gap at $\nu = 7/2$, the same type of analysis [21] cannot be applied to the present sample. Nonetheless we think that had the disorder been the cause of the anomaly observed at $\nu = 5/2$, a similar effect would also be present at $\nu = 7/3$. However, as seen in Fig.4, the density dependence of the gaps at $\nu = 5/2$ and $\nu = 7/3$ are very different. Furthermore, the smooth density dependence of the mobility shown in Fig.2b strengthens the argument that disorder effects are not driving the transition seen in Fig.4 at $\nu = 5/2$.

Finally we note that due to the combined effects of Landau level mixing and disorder, the energy gap at $\nu = 5/2$ is expected to close at the lowest densities. At this filling factor we report a measurable gap down to the density of $n = 4.90 \times 10^{10}$ cm^{-2}. Furthermore, the linear extrapolation of our data from Fig.4 shows that the energy gap at $\nu = 5/2$ vanishes near $n \simeq 4.5 \times 10^{10}$ cm^{-2}. This value is in a reasonable agreement with a recent extrapolation of data collected at higher electron densities [34].

In conclusion, we have studied the density dependence of the fractional quantum Hall state at $\nu = 5/2$ in the regime of extremely large LLM. We have observed an anomaly in the density dependence of the energy gap of this state. The observed anomaly is consistent with expectations of a topological phase transition between the Pfaffian and anti-Pfaffian ground states at $\nu = 5/2$. We also analyze other possible origins of the observed anomaly in the $\nu = 5/2$ fractional quantum Hall state but find that a spin transition, effects of the second electric subband in the confining potential, and the effects of disorder are unlikely to account for our observations.

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